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A Study on Cost Optimised Structural Design of Reinforced Concrete Beams

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Abstract— The objective of every Structural Engineer is to design the optimum structural systems. Through intuition, experience, and repeated trials, this has been achieved traditionally. This solutions may not be the best among the several possible alternatives. With optimisation techniques, the designer can evaluate more alternatives, thus resulting in a better and more cost-effective design. This paper aims to find out the optimum design of a singly reinforced rectangular Reinforced Concrete (RC) beam for a given imposed load subject to codal and practical constraints. The cost of the beam can be expressed as a function of the quantity of concrete and steel, grade of concrete, size of form work etc. This function will be the objective function for the problem. The beam should satisfy the strength and serviceability conditions as per the design code IS 456 which will act as constraints for the optimisation problem. The objective of the optimisation is to minimize the total cost of beam subject to the constraints.

Index Terms— Genetic Algorithm, Constraints, IS 456, Objective function, Optimum cost, MATLAB, RCC Beam, Structural design, Structural Optimisation.

1 Introduction

OPTIMISATION is the art of choosing an alternative with the most cost effective or highest achievable performance from several possible alternatives under the given constraints, by maximizing the desired factors and minimizing the undesired factors.

Cost optimisation of Reinforced Cement Concrete (RCC) Beam is a favorite problem of many researchers working in structural optimisation. Babiker *et al.* [1] used a model based on Artificial Neural Networks to perform the cost optimization of simply supported beams by including the cost of concrete, the cost of reinforcement and the cost of formwork. The beams were designed according to the requirements of the American Concrete Institute (ACI) standard ACI 318-08.

Most of the recent works on this topic was carried out using Genetic Algorithm (GA) technique of optimisation. Yousif and Najem [2] presented the application of genetic algorithms (GA) for the optimum cost design of RCC continuous beams based on the specifications of the ACI 318-08. The results of the illustrated example problem gave rational, reliable, economic, and practical designs. A comparative study between one of the classical optimization techniques, Generalized Reduced Gradient (GRG) and one of the heuristic techniques, Genetic Algorithm was carried out by Ismail [3]. The comparison revealed the superiority of the GA over the classical GRG. Bhalchandra and Adsul [4] also showed the superiority of the GA technique over the GRG and Interior Point optimization technique. The optimum design of simply supported doubly reinforced beams with uniformly distributed and concentrated load has been done by incorporating actual self weight of beam in the problem.

An attempt has been made by Alex and Kottalil [5], [6] to demonstrate the application of the GA to the design of reinforced concrete cantilever and continuous beams. The guidelines given by the Indian Standard, IS 456 was used as the basis for the design. Cost optimisation was carried out to get the most economical concrete section and the reinforcements at user defined intervals. Prakash *et al.* [7] studied economical aspects in the design of reinforced concrete beams. RCC Rectangular and Flanged sections were designed manually and using MS-Excel program. The Singly reinforced, Doubly reinforced, and Flanged sections were designed as per IS 456-2000 codal provisions for a constant Imposed Load of 25 kN/m. Different spans and depth to breadth ratios were also considered in the study.

In most of the works the design was based on international standards like ACI. The works based on Indian standards were found less. Also the design variables chosen for the optimisation problems varied from researcher to researcher. This paper aims to find out the optimum design of a singly reinforced rectangular RC beam for a given imposed load subject to codal and practical constraints. The cost of the beam can be expressed as a function of the quantity of concrete and steel, grade of concrete, size of form work etc. This function will be the objective function for the problem. The beam should satisfy the strength and serviceability conditions as per the design code IS 456, which will act as constraints for the optimisation problem. The objective of the optimisation is to minimize the total cost of beam subject to the constraints.

2 THE Objective Function

The criterion with respect to which the design is optimised, when expressed as a function of design variables is known as the objective function. An optimisation problem may have one or more objective functions. The objective of this problem is to minimize the total cost of the beam. The total

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cost of the beam is the sum of cost of concrete, steel and formwork. The objective function is given by,

Total cost, $TC = Cc\ (bD - A_{st}) + C_s\ A_{st} + C_f\ (b + 2D)$ (1)

After converting all the terms into single unit, the equation becomes,

$$TC = C_c\ (bD-A_{st})\ x\ 10^{-6} + C_s\ A_{st}\ x\ 7850\ x\ 10^{-6} + C_f\ (b+2D)\ x\ 10^{-3} \quad (2)$$

Where,

C_c: Cost of concrete (□/m^3)
C_s: Cost of reinforcement (□/kg)
C_f: Cost of formwork (□/m^2)
D: Overall depth (mm)
b: Breadth (mm) and
A_{st}: Area of steel

Fig. 1 shows the cross section of a singly reinforced rectangular RCC beam including formwork. The grade of steel in the present study was chosen as Fe415.

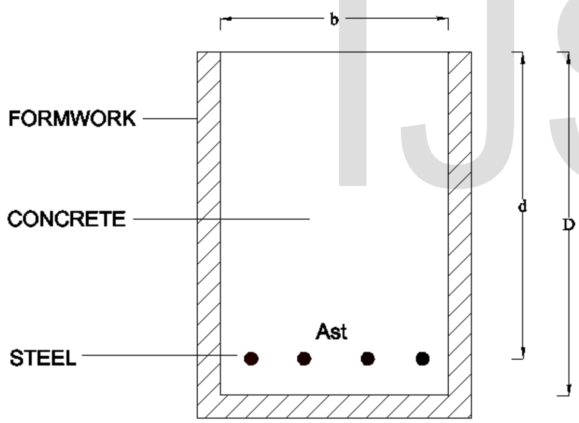


Fig. 1. Cross section of RCC beam

3 Constraints

The design of singly reinforced concrete beam is to be done by limit state method as per the specifications of IS 456:2000 [8]. The design criteria as per the code will act as the constraints for the problem.

3.1 Control of Deflection

For a simply supported beam the vertical deflection limits will be satisfied if the span to depth ratio is kept below 20. Equation (3) gives the first constraint for the optimization problem.

$$\frac{l}{d} \leq 20 \quad (3)$$

Where l and d are the effective span and effective depth respectively.

3.2 Minimum and maximum Reinforcement

IS 456 recommends a minimum quantity of tension reinforcement to ensure a minimum ductility to the concrete and thereby prevents sudden failures. The maximum percentage of reinforcement is limited to 4% to eliminate the chance of formation of voids during the placing of concrete. Equation (4) and (5) are the constraints that satisfies the above two requirements.

$$\frac{A_{st}}{bD} \leq 0.04 \quad (4)$$

$$\frac{A_{st}}{bd} \geq \frac{0.85}{f_y} \quad (5)$$

Where f_y is the characteristic strength of steel reinforcement. For Fe415 grade steel, f_y = 415 MPa.

3.3 Limiting Moment

For a beam section to be designed as singly reinforced, the factored Bending Moment (BM) should be with in the moment capacity of the section. Otherwise it should be designed as doubly reinforced. The constraint to ensure this criteria is (7).

$$M_u \leq M_{ulim} \quad (6)$$

For Fe415 grade steel,

$$M_u \leq 0.138\, f_{ck}\, b\, d^{\,2} \quad (7)$$

Where M_u is the factored BM, M_{ulim} is the limiting moment and fck is the characteristic strength of concrete.

3.4 Shear Strength

The shear stress to be taken by stirrup reinforcement is the difference of the nominal shear stress and the design shear strength of concrete without shear reinforcement. This will act as an equality constraint for the problem.

$$(\tau_v - \tau_c)\ bd\ = \frac{0.87\, f_y\ A_{sv}\ d}{s_v} \quad (8)$$

Nominal shear stress is to be calculated using (9).

$$\tau_v = \frac{V_u}{bd} \quad (9)$$

Shear strength of concrete without shear reinforcement (τ_c) is given by the empirical formula,

$$\tau_c = \frac{0.85\ \sqrt{0.8 f_{ck}}\ (\sqrt{1+5\beta} - 1)}{6\beta} \quad (10)$$

$\beta = \frac{0.8\, f_{ck}}{6.89\, p_t}$ or 1, whichever is bigger and $p_t = \frac{100\, A_{st}}{bd}$

Where A_{sv} is the area of shear reinforcement p_t is the percentage of tension reinforcement.

3.4 Maximum Spacing of Stirrups

The maximum spacing of stirrups (s_v) should be 75% of effective depth or 300 mm, whichever is smaller. There fore 0.75 d and 300 mm will be the upper bound values for s_v (10), (11).

$$s_v \leq 0.75d \tag{10}$$

$$s_v \leq 300 \tag{11}$$

In addition to the above codal constraints, the maximum breadth of beam is set not to exceed its depth with both of them having a lower bound of 200 mm. Also the maximum depth of beam is limited to 750 mm to eliminate the provision of side face reinforcement. The spacing of stirrup was set not to go below 60 mm. The lower bound of A_{st} was kept 100 mm² which is equivalent to two 8 mm bars.

4 The Design Variables and Constants

There are 5 design variables chosen for the optimisation problem. Four of them are continuous variables which are breadth of the beam *b*, effective depth *d*, area of tension steel A_{st} and spacing of stirrups s_v. The last variable is the characteristic strength of concrete f_{ck} with discrete set of values. The grades under consideration are M15, M20, M25 and M30. The corresponding characteristic strengths are 15, 20, 25, and 30 N/mm².

The constant parameters used in the problem include *Effective cover d'*, *Cost of concrete* C_c, *Cost of reinforcement* C_s, *Cost of formwork* C_f and *Area of stirrups* A_{sv}. The value of *Effective cover* was assumed as 40 mm for all the cases. The shear reinforcement used was 2-legged stirrups of 8 mm diameter.

5 Cost of Materials

The cost of materials were taken from Delhi Schedule of Rates 2014 [9].

1. Cost of TMT bars including cost of steel, cost of straightening, cutting, bindings and placing in position,
   C_s = 68.1 ₹/kg
   Density of steel = 7850 kg/m³
2. Cost of reinforced cement concrete work in beams excluding cost of centering shuttering, finishing and cost of reinforcement Cc are as given in Table 1.
3. Cost of formwork including centering shuttering and removal,
   C_f = 360.8 ₹ /m²

TABLE 1
COST OF CONCRETE

Sl. No.	Grade	Cost (□/m³)
1	M15	6570.00
2	M20	7074.30
3	M25	8634.05
4	M30	8710.55
5	M35	8787.00

6 Methodology

The prime objective of the problem is to optimise the total cost of the beam confirming to the listed constraints. Every optimisation problem requires the selection of a suitable optimisation algorithm. In this problem the Objective function is a function of 3 continuous valued design variables b, d, Ast and a discrete variable Cc. There are both linear and nonlinear constraints. Hence the problem is a non linear constrained optimisation with mixed variables. Genetic Algorithms (GA) are well suited for solving such problems.

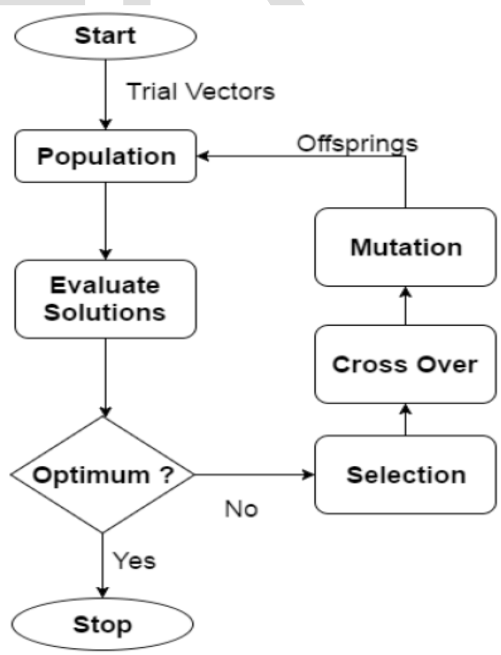


Fig. 2. Flowchart of genetic algorithm

Genetic algorithm is a modern optimisation technique that developed by inspiration from the theory of natural genetics. In conventional optimisation techniques, iterations are done by choosing a single trial design point. This may lead to the trapping of solution in a region of local minimum. Usually the result may be in the vicinity of initial point. In GA, a population of points called trial design vectors is used to start the procedure instead of a single design point. Hence it is less likely to get trapped at local minima. GA can solve variety of problems that are not suited for conventional methods, like problems involving discontinuous, nonlinear and undifferentiable functions [10].

The optimisation was done by coding in Matlab using its genetic algorithm solver function 'ga' - the genetic algorithm solver for mixed-integer or continuous-variable optimization, constrained or unconstrained [11].

7 Results and Discussions

The program was executed for different values of imposed loads and spans. The spans chosen was 2, 3 4, 6 and 8 meters. The imposed loads were varied as 10, 25, 50 and 100 kNs. The design variables were found increasing with increase in imposed load and span and showed a good trade off between the grade of concrete and the sectional properties.

The cost of concrete increases from M15 to M30. This may be because rich mixes require additional ingredients like admixtures, more specific procedures and testing. However there is no big difference among the rich mixes. M15, M20, M25 and M30 were respectively 6570.00, 7074.30, 8634.05 and 8710.55 rupees. Optimum cost of concrete will be achieved if lower grades of concrete are used. To meet the strength constraints, the sectional properties should be accordingly increased. Fig. 3 shows the variation of grade of concrete with span. The results shows that the program tried to maximize the use of M15 grade for shorter spans. For bigger spans the bending moments will be higher and hence requires higher grades to resist the same.

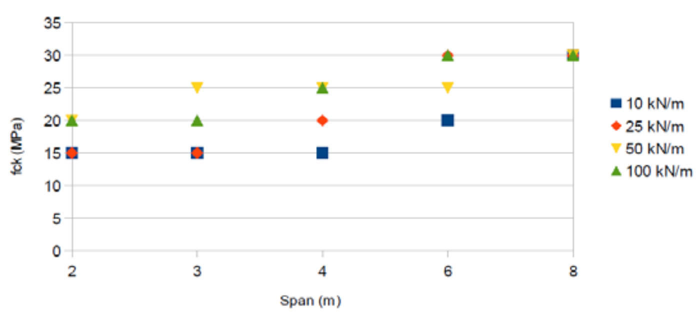
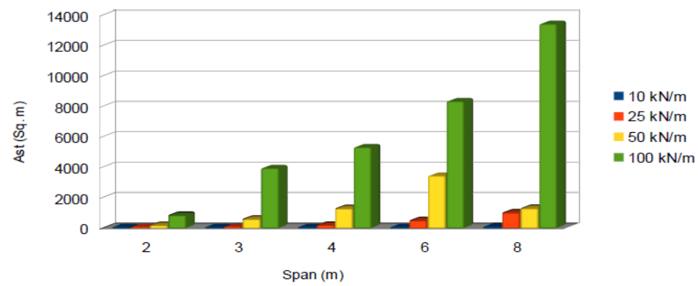


Fig. 3. Span vs Grade of concrete

The cost of reinforcing steel is around 20 times of that of cost of concrete. Therefore the main focus of Renforced Concrte beam optimisation should be on the minimization of quantity of steel. The program has tried to keep the area of steel closer to the lower bound value as far as possible The area of steel obtained was very less even for higher spans. How ever at a heavy load of 100 kN/m^2, the A_{st} values were found higher. Fig. 4 and 5 shows the variation of



span with Area of steel and Depth of beam respectively.

Fig. 4. Span vs Area of steel

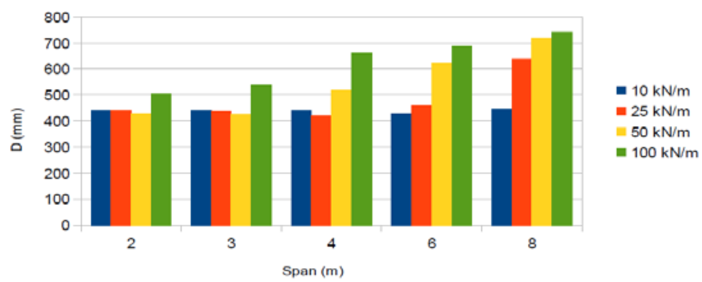


Fig. 5. Span vs Depth of beam

For smaller spans the A_{st} values are nearer to the lower bound value. Also the grade of concrete were lesser ones. The limiting moment has a second degree variation with respect to effective depth. This may be attributed as the reason for the selection of higher depths even for smaller spans. The spacing of stirrups s_v values were chosen closer to the upper bound value 300 mm for almost all the spans. However for heavy loading, the spacing has gone down. The variation of stirrup spacing with span is shown in the Fig. 6.

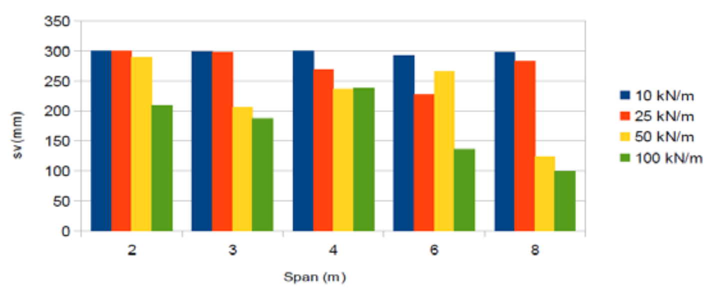


Fig. 6. Span vs stirrup spacing

In practical problems there may be additional site specific constraints like architectural limits for beam dimensions, Available size of form work, Available grade of concrete and steel etc. The program can be conveniently modified to suit the local requirements. The design variables b, d, Ast were considered as continuous valued. These variables can be discretized to decrease the size of population for the genetic algorithm. This may lead to more realistic results. The area of shear reinforcement is kept constant here. It is better to introduce it as a variable in the problem.

8 Conclusion

The optimisation of singly reinforced RCC beam was carried out by the minimisation of cost of the beam considering the design constraints. The problem was solved using genetic algorithm optimisation technique. Matlab R2014a was used for the programming of the optimisation problem. The genetic algorithm based optimisation gave reasonable results satisfying the design constraints. The results show a good trade off between the grade of concrete and the sectional properties. The program can be suitably modified to meet the practical requirements of a given problem. The results obtained from the program can be used as an aid for design of RCC members.

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